Review of Introduction to Probability and Statistics

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Homework #2

1. Consider an untested batch of memory chips that have a known failure rate of 8% (yield = 92%).

    a. What is the probability that exactly 4 out of a lot of 12 will fail?

    b. What is the probability that at most 4 out of a lot of 12 will fail?

    c. What assumptions have you made in answering the two above questions?

2. Consider a discrete random variable *X* with the following PMF:

$$p_X(x) = \begin{cases} \frac{1}{9}, & if\ x\ is\ an\ integer\ in\ the\ range\ of\ [-4,4] \\ 0, & otherwise \end{cases}$$

    Determine the PMF for $Y = |X|$.

3. You are at a party with 500 guests. What is the probability that exactly one other guest has the same birthday as you? For simplicity, exclude the possibility of a February 29 birthday. After calculating the result exactly, approximate the answer by using a Poisson distribution.

4. After renting a large house on vacation, the realtor gave you five unmarked and similar-looking keys for the five doors of the house. Find the PMF of the number of trials required before successfully opening the front door under these different assumptions:

    a. After an unsuccessful trial, you set the bad key aside so you don't try it again.

    b. After an unsuccessful trial, you put the bad key back into the pool of keys so that each key has an equal likelihood of being picked.

5. Let X be a discrete random variable with PMF

$$p_X(x) = \begin{cases} x^2/a, & if\ x = -3,-2,-1,0,1,2,3 \\ 0, & otherwise \end{cases}$$

    a. Find *a* and E[X].
    b. What is the PMF of the random variable $Z = (X - E[X])^2$?
    c. Using the result from part (b), find the variance of X.
    d. Find the variance o f X using the formula $var[X] = \sum_{all\ x}(x - E[X])^2\ p_X(x)$.

6. A city's temperature is modeled as a random variable with mean and standard deviation both equal to 10 degrees Celsius. A day is described as "typical" if the temperature during that day ranges within one standard deviation from the mean. What would be the temperature range for a typical day if temperature were expressed in degrees Fahrenheit?

7. The St. Petersburg paradox: You toss independently a fair coin and you count the number of tosses until the first tail appears. If this number is n, you receive 2^n dollars. What is the expected amount that you will receive? How much would you be willing to pay to play this game?

8. Alice passes through four traffic lights on her way to work, and each light is equally likely to be green or red, independent of the others.
    a. What is the PMF, the mean, and the variance of the number of red lights that Alice encounters?
    b. Suppose that each red light delays Alice by exactly two minutes. What is the variance of Alice's commuting time?

9. Given the PDF for a uniformly distributed continuous random variable X shown below, derive the expectation and the variance of X.

$$f_X(x) = \begin{cases} 1/(b-a), & if\ a < x < b \\ 0, & otherwise \end{cases}$$

10. What is the CDF (cumulative distribution function) for the PDF of problem 9?

11. Consider the exponential PDF with (positive) parameter λ:

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x}, & if\ x \geq 0 \\ 0, & otherwise \end{cases}$$

    a. Show that this is a legitimate PDF.
    b. Find the expectation and variance of *X*.

12. Let X and Y be normal random variables with means 0 and 1, respectively, and variances 1 and 4, respectively.

    a. Find $\mathbb{P}(X < 1.5)$ and $\mathbb{P}(X < -1)$.
    b. Find the PDF of $(Y - 1)/2$.
    c. Find $P(-1 < Y < 1)$.

13. A city's temperature is modeled as a normal random variable with mean and standard deviation both equal to 10 degrees Celsius. What is the probability that the temperature at a randomly chosen time will be less than or equal to 59 degrees Fahrenheit?