The Rayleigh Depth of Focus

Chris A. Mack, *KLA-Tencor, FINLE Division, Austin, Texas*

John William Strutt, the third Baron Rayleigh of Terling Place, was one the most celebrated physicists of his day [1]. Hardly a subject in classical physics went without a contribution from this prolific scientist, including the discovery of argon, for which he won the Nobel prize in 1904. Although his contributions in acoustics, mathematics, optical scattering, and hydrodynamics were very significant, he is most remembered by lithographers for his treatment of imaging. Describing the imaging capabilities of telescopes, Lord Rayleigh developed the now famous Rayleigh criteria for resolution and depth of focus. In this column we'll repeat a version of Lord Rayleigh's derivation (using the modern terminology and use case of lithography) and see how the Rayleigh depth of focus criterion can be extended to high numerical apertures and immersion lithography [2].

A common way of thinking about the effect of defocus on an image is to consider the defocusing of a wafer as equivalent to causing an aberration – an error in curvature of the actual wavefront relative to the desired wavefront (i.e., the one that focuses on the wafer – see Figure 1). Looking at Figure 1b, the distance from the desired to the "defocused" wavefront goes from zero at the center of the exit pupil and increases as we approach the edge of the pupil. This distance between wavefronts is called the *optical path difference* (OPD). The OPD is a function of the defocus distance δ and the position within the pupil and can be obtained from the geometry shown in Figure 2. Describing the position within the exit pupil by an angle θ, the optical path difference is given (after a bit of geometry and algebra) by

$$OPD = \delta(1-\cos\theta) \tag{1}$$

Depth of focus (DOF) is defined generically as the range of focus that can be tolerated. While an exact criterion for "tolerated" is application dependent, a simple example can be used to guide a basic description of DOF. Consider the imaging of an array of small lines and spaces. The diffraction pattern for such a mask is a set of discrete diffraction orders, points of light entering the lens spaced regularly depending only on the wavelength of the light λ and the pitch *p* of the mask pattern. The angles at which these diffraction orders will emerge from the lens are given by Bragg's condition:

$$\sin\theta = \frac{m\lambda}{p} \tag{2}$$

where *m* is an integer. Using this integer to name the diffraction orders, a high resolution pattern of lines and spaces will result in only the zero and the plus and minus first diffraction orders passing through the lens to forming the image.

Combining equations (1) and (2) we can see how much *OPD* will exist between the zero and first orders of our diffraction pattern. Unfortunately, some trigonometric manipulations will be required to convert the cosine of equation (1) into the more convenient sine of equation (2). One such manipulation uses a Taylor series:

$$OPD = \delta(1-\cos\theta) = \tfrac{1}{2}\delta\left(\sin^2\theta + \frac{\sin^4\theta}{4} + \frac{\sin^6\theta}{8} + \ldots\right) \tag{3}$$

At the time of Lord Rayleigh, lens numerical apertures were relatively small. Thus, the largest angles going through the lens were also quite small and the higher order terms in the Taylor series could be ignored, giving

$$OPD \approx \frac{1}{2}\delta\sin^2\theta \tag{4}$$

How much OPD can our line/space pattern tolerate? Consider the extreme case. If the *OPD* were set to a quarter of the wavelength, the zero and first diffracted orders would be exactly 90º out of phase with each other. At this much *OPD*, the zero and first orders would not interfere with each other at all and no pattern would be formed. The true amount of tolerable *OPD* must be less than this amount.

$$OPD_{\max} = k_2\frac{\lambda}{4}, \quad where\ k_2 < 1 \tag{5}$$

Substituting this maximum permissible *OPD* into equation (4), we can find the DOF.

$$DOF = 2\delta_{\max} = k_2\frac{\lambda}{\sin^2\theta} \tag{6}$$

At this point Lord Rayleigh made a crucial application of this formula that is often forgotten. While equation (6) would apply to any small pattern of lines and spaces (that is, any pitch applied to equation (2) so that only the zero and first orders go through the lens), Lord Rayleigh essentially looked at the extreme case of the smallest pitch that could be imaged – the resolution limit. The smallest pitch that can be printed would put the first diffracted order at the largest angle that could pass through the lens, defined by the numerical aperture, *NA*. For this one pattern, the general expression (6) becomes the more familiar and specific Rayleigh DOF criterion:

$$DOF = k_2\frac{\lambda}{NA^2} \tag{7}$$

From the above derivation we can state the restrictions on this conventional expression of the Rayleigh DOF: relatively low numerical apertures imaging a binary mask pattern of lines and spaces at the resolution limit. To lift these restrictions we simply use the exact OPD expression and leave the angle to be defined by equation (2).

$$DOF = \frac{k_2}{2}\frac{\lambda}{(1-\cos\theta)} = \left(\frac{k_2}{4}\right)\frac{\lambda}{\sin^2\left(\frac{\theta}{2}\right)} \tag{8}$$

This high NA version of the Ralyeigh DOF criterion still assumes we are imaging a small binary pattern of lines and spaces, but is appropriate at any numerical aperture. It can also be modified to account for immersion lithography quite easily. When the space between the lens and the wafer is filled with a fluid of refractive index n_{fluid}, the optical path difference becomes the physical path different multiplied by this refractive index. Thus equation (1) becomes

$$OPD = n_{fluid}\ \delta(1-\cos\theta) \tag{9}$$

and the high NA version of the Rayleigh criterion becomes

$$DOF = \frac{k_2}{2}\frac{\lambda}{n_{fluid}\,(1-\cos\theta)} \tag{10}$$

Likewise, the angle θ can be related to the pitch by the modification of equation (2) to account for immersion.

$$n_{fluid}\sin\theta = \frac{m\lambda}{p} \tag{11}$$

Combining equations (10) and (11) one can see how immersion will improve the depth of focus of a given feature:

$$\frac{DOF(immersion)}{DOF(dry)} = \frac{1-\sqrt{1-(\lambda/p)^2}}{n_{fluid}-\sqrt{(n_{fluid})^2-(\lambda/p)^2}} \tag{12}$$

As Figure 3 shows, the improvement in DOF is at least the refractive index of the fluid, and grows larger from there for the smallest pitches. It’s no wonder immersion lithography is attracting so much attention.

References

1. http://www.nobel.se/physics/laureates/1904/strutt-bio.html

2. B. J. Lin, “The k_3 Coefficient in Nonparaxial λ/NA Scaling Equations for Resolution, Depth of Focus, and Immersion Lithography,” *Journal of Microlithography, Microfabrication, and Microsystems*, Vol. 1, No. 1 (April 2002) pp. 7-12.

Lord Rayleigh

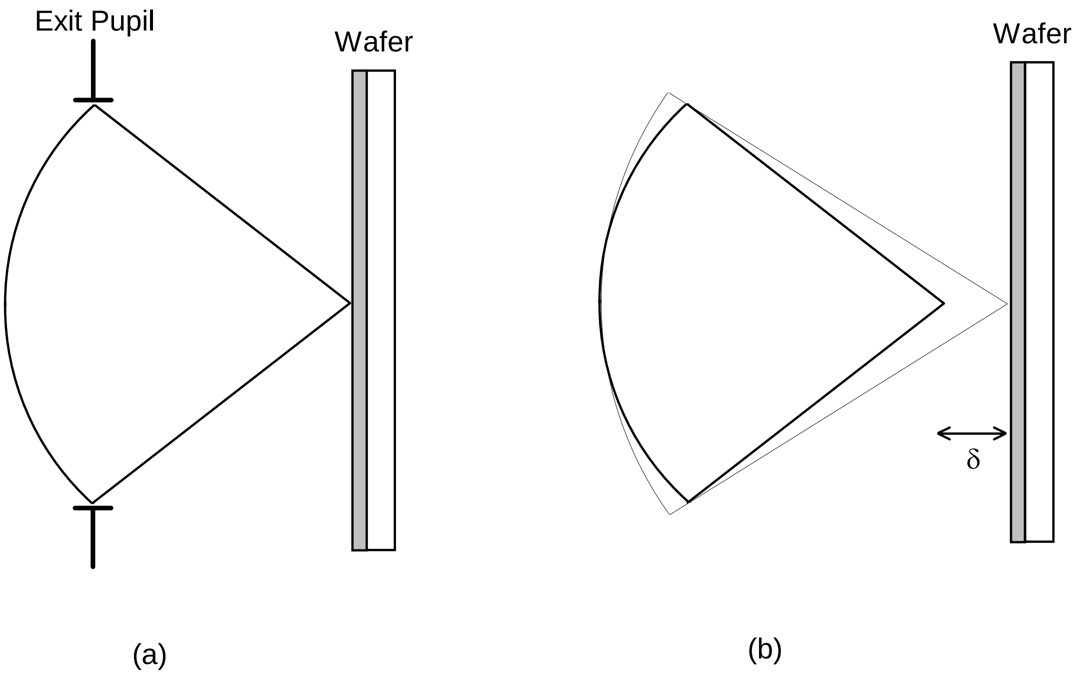

Figure 1. Focusing of light can be thought of as a converging spherical wave: a) in focus, and b) out of focus by a distance δ.

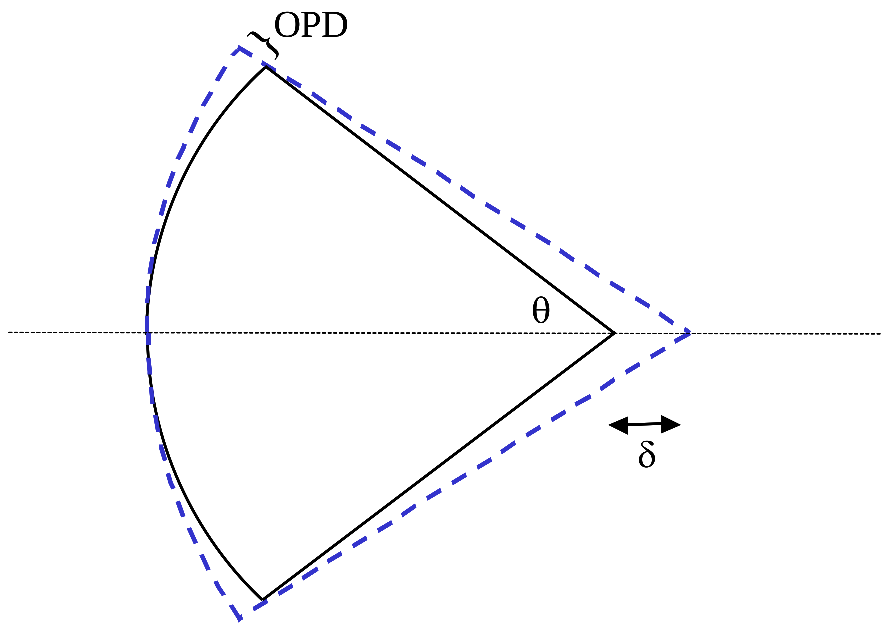

Figure 2. Geometry relating the optical path difference (OPD) to the defocus distance δ and the angle θ.

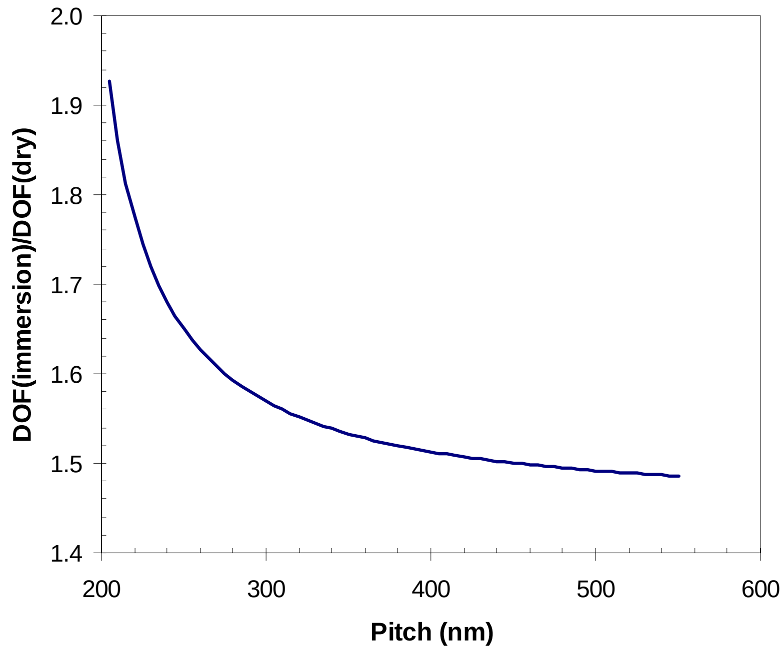

Figure 3. For a given pattern of small lines and spaces, using immersion improves the depth of focus by at least the refractive index of the fluid (in this example, λ = 193nm, n_{fluid} = 1.46).